Definition of Extrema

Let f be defined on an interval I containing c.

1. $f(c)$ is the **minimum of f in I** if $f(c) \le f(x)$ for all x in **I**.
2. $f(c)$ is the **maximum of f in I** if $f(c) \ge f(x)$ for all x in **I**.

The minimum or maximum of a function on an interval can also be called extrema, absolute minimums, or absolute maximums.

Extreme Value Theorem

If ***f*** is continuous on a closed interval $[a,b]$, then ***f*** has both a maximum and a minimum on the interval.

Note: A function need not have a maximum or a minimum on an interval. See picture:

Definition of Relative Extrema

1. If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of ***f***.
2. If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of ***f***.

Definition of Critical Number

Let ***f*** be defined at c. If $f'(c) = 0$ or if f' is undefined at c, then c is a **critical number** of ***f***.

Relative Extrema Occur Only at Critical Numbers.

If ***f*** has a relative minimum or relative maximum at **x=c**, then c is a critical number of ***f***. (Note that the converse is not necessarily true… critical numbers need not produce relative extrema.)

Guidelines for Finding Extrema on an Interval

1. Find critical numbers of ***f*** in $[a,b]$ … setting $f'(c) = 0$.
2. Evaluate ***f*** at each critical value in $[a,b]$.
3. Evaluate ***f*** at each endpoint of $[a,b]$.
4. The least of these is the minimum; the greatest is the maximum.

Pg. 169 - Problems

13-18 Find any critical numbers of the function

14. $g(x) = x^2(x^2 - 4)$

16. $f(x) = \dfrac{4x}{x^2 + 1}$

18. $f(\vartheta) = 2\sec\vartheta + \tan\vartheta$, $0 \le \vartheta < 2\pi$

#19-36 Determine the absolute extrema of the function and the x-value in the interval where it occurs.

20. $f(x)=\frac{2}{3}x+\frac{5}{3}$ on $[0,5]$

22. $f(x)=x^2+2x-4$ on $[-1,1]$

27. $g(t)=\frac{t^2}{t^2+3}$ on $[-1,1]$

34. $g(x)=\sec x$ on $\left[\frac{-\pi}{6},\frac{\pi}{3}\right]$

60. A retailer has determined that the cost C of ordering and storing x units of a certain product is

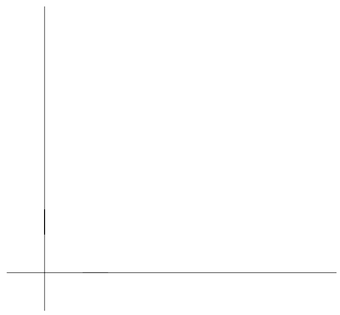
$$C=2x+\frac{300{,}000}{x}\ ,\quad 1\le x\le 300$$

The delivery truck can bring at most 300 units per order. Find the order size that will minimize cost. Could the cost be decreased if the truck were replaced with one that could bring at most 400 units? Explain.

Notes Section 3.2 MEAN VALUE THEOREM

Rolle's Theorem

Let *f* be continuous on the closed interval $[a,b]$ and differentiable on the open interval $f'(c)=0$. If $f(a)=f(b)$ then there is at least one number c in (a,b) such that $f'(c)=0$.



Case #1: $f(x)$ is constant and $f'(x)=0$ for all **x** in (a,b).

Case #2: $f(x)>d$ at some place in (a,b). By the extreme value theorem, there is a maximum at some c in the interval. Since $f(c)>d$, the maximum does not occur at an endpoint. Therefore, $f(x)$ has a maximum in open (a,b). Therefore, $f(c)$ is a relative maximum. Thus *c* is a critical number of $f(x)$. Because *f* is differentiable at *c*, we can conclude that $f'(c)=0$.

Case #3: $f(x)<d$ at some point in (a,b). Use similar reasoning as in case 2 but involving a minimum instead of a maximum.

The Mean Value Theorem

If *f* is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), then there exists a number c in (a,b) such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}.$$

The "mean" in the Mean Value Theorem refers to the mean (or average) rate of change of *f* in the interval $[a,b]$.

Geometrically, the theorem guarantees the existence of a tangent line that is parallel to the secant line through the points $(a,f(a))$ and $(b,f(b))$.

The equation of the secant line is

$$y=\frac{y_2-y_1}{x_2-x_1}(x-x_1)+y_1$$

$$y=\left[\frac{f(b)-f(a)}{b-a}\right](x-a)+f(a)$$

Let $g(x)=f(x)-y$. Then the difference between $f(x)$ and **y** is

$$g(x)=f(x)-\left[\frac{f(b)-f(a)}{b-a}\right](x-a)-f(a)$$

There exists a *c* in (a,b) such that $g'(c)=0$.

$$0=g'(c)=f'(c)-\frac{f(b)-f(a)}{b-a}=0$$

There exists a *c* such that $f'(c)=\frac{f(b)-f(a)}{b-a}$.

In terms of the rate of change, the MVT implies that there must be a point in the open interval (a,b) at which the instantaneous rate of change is equal to the average rate of change over the interval $[a,b]$.

Alternate form of the MVT

$$f(b)=f(a)+(b-a)f'(c)$$

pg. 176 - problems

In exercises 11-24, determine whether Rolle's Theorem can be applied to *f* on the indicated interval. If Rolle's theorem can be applied, find the values of c in the interval such that $f'(c)=0$.

14. $f(x)=(x-3)(x+1)^2$ **on** $[-1,3]$

16. $f(x) = 3 - |x - 3|$ on $[0,6]$

18. $f(x) = \dfrac{x^2 - 1}{x}$ on $[-1,1]$

21. $f(x) = \dfrac{6x}{\pi} - 4\sin^2 x$ on $\left[0, \dfrac{\pi}{6}\right]$

In exercises 39-46, apply the MVT to *f* on the indicated interval. In each case, find all values of c in the interval (a,b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

40. $f(x) = x\left(x^2 - x - 2\right)$ on $[-1,1]$

46. $f(x) = 2\sin x + \sin 2x$ on $[0,\pi]$

70. Find a function f that has the derivative $f'(x) = 4$ and whose graph passes through the point (0,1)

The Mean Value Theorem

to the tune "From the Halls of Montezuma"

If a function is continuous
On a closed set "a" to "b"
And it's also differentiable
On the open set "a" "b,"
You can always find a "c" inside
Such that f prime at point C
Is equivalent to just the slope
Of the line from A to B.