Definitions:

A function f is **increasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function f is **decreasing** on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b).

1. If $f'(x) > 0$ for all x in (a,b), then f is increasing on $[a,b]$.
2. If $f'(x) < 0$ for all x in (a,b), then f is decreasing on $[a,b]$.
3. If $f'(x) = 0$ for all x in (a,b), then f is constant on $[a,b]$.

Guidelines for Finding Intervals of Increasing or Decreasing Behavior

1. Locate critical numbers, setting $f'(x) = 0$.
2. Create intervals based on the critical numbers and determine the sign of $f'(x)$ in each of these intervals.

If a function is **strictly increasing** or **strictly decreasing** on an interval, we say that it is **strictly monotonic**.

The First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a **relative minimum** of f on I.
2. If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a **relative maximum** of f on I.

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In exercises 17-38, find the critical numbers of f (if any), find the open intervals on which the algebraic function is increasing or decreasing, and locate all relative extrema.

24. $f(x) = (x+2)^2(x-1)$

28. $f(x) = x^{2/3} - 4$

40. Find the open intervals on which $\boldsymbol{f(x) = \sin x \cos x}$ is increasing or decreasing and locate all relative extrema.

86. The function $s(t) = t^2 - 7t + 10$ describes the motion of a particle moving along a line.

a. Find the velocity function of the particle at any time $t \geq o$.
b. Identify the time interval(s) when the particle is moving in a positive direction
c. Identify the time interval(s) whent eh particle is moving in a negative direction
d. Identify the time(s) when the particle changes direction.

Section 3.4 CONCAVITY & THE SECOND DERIVATIVE TEST

Definition of Concavity

Let ***f*** be differentiable on an open interval ***I***. The graph of ***f*** is **concave upward** on I if f' is increasing on the interval and **concave downward** on ***I*** if f' is decreasing on the interval.

Test for Concavity

Let ***f*** be a function whose second derivative exists on an open interval ***I***.

1. If $f''(x) > 0$ for all x in ***I***, then the graph of ***f*** is concave upward in ***I***.
2. If $f''(x) < 0$ for all x in ***I***, then the graph of ***f*** is concave downward in ***I***.

A graph changes concavity at a **point of inflection**. At these points $(c, f(c))$, $f''(x)$ is either 0 or undefined. (The converse is not necessarily true.. sometimes $f''(x) = 0$ at places that are not points of inflection.)

The Second Derivative Test

Let ***f*** be a function such that $f'(c) = 0$ (hence having relative extrema at $x = c$) and the second derivative of ***f*** exists on an open interval containing c.

1. If $f''(c) > 0$, then $f(c)$ is a relative minimum.
2. If $f''(c) < 0$, then $f(c)$ is a relative maximum.
3. If $f''(c) = 0$, then the test fails. In such cases, you can use the first derivative test.

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In exercises 27-40, find all relative extrema. Use the Second Derivative Test where applicable.

28. $f(x) = x^2 + 3x - 8$

36. $f(x) = \sqrt{x^2 + 1}$

40. $f(x) = 2\sin x + \cos 2x, \quad 0 \le x \le 2\pi$

In exercises 11-26, find all relative extrema and points of inflection. Discuss concavity.

12. $f(x) = 2x^3 - 3x^2 - 12x + 5$

18. $f(x) = x\sqrt{x+1}$

22. $f(x) = 2\csc\frac{3x}{2}, \quad 0 < x < 2\pi$

62. Find a, b, c, and d such that the cubic $f(x) = ax^3 + bx^2 + cx + d$ satisfies the indicated conditions.

Relative maximum: $(2,4)$

Relative minimum: $(4,2)$

Inflection point: $(3,3)$

Section 3.5 LIMITS AT INFINITY

Definitions:

1. The statement $\lim_{x \to \infty} f(x) = L$ means that for each $\varepsilon > 0$ there exists an **M>0** such that $|f(x) - L| < \varepsilon$ whenever **x>M.**
2. The statement $\lim_{x \to -\infty} f(x) = L$ means that for each $\varepsilon > 0$ there exists an **N<0** such that $|f(x) - L| < \varepsilon$ whenever **x<N.**

picture:

Definition of Horizontal Asymptotes

The line **y=L** is a **horizontal asymptote** of the graph of ***f*** if $\lim_{x \to -\infty} f(x) = L$ or $\lim_{x \to \infty} f(x) = L$.

Limits at Infinity

If r is a positive rational number and c is any real number, then $\lim_{x \to \infty} \frac{c}{x^r} = 0$. Furthermore, if x^r is defined when **x<0**, then $\lim_{x \to -\infty} \frac{c}{x^r} = 0$.

The limit that results $\frac{\infty}{\infty}$ is called an **indeterminate form.** To resolve this problem, you can divide both the numerator and denominator by x. The limit can then be evaluated.

Example:

Find $\lim_{x \to \infty} \left(\frac{2x - 1}{x + 1}\right)$

Guidelines for Rational Functions

1. If the degree of the numerator is *less* than the degree of the denominator, then the x-axis is a horizontal asymptote and the limit as x approaches infinity of the rational function is **0.**
2. If the degree of the numerator *equals* the degree of the denominator, then the limit as x approaches infinity of the rational function is the **ratio of the leading coefficients.**
3. If the degree of the numerator is *greater* than the degree of the denominator, then there is no horizontal asymptote and the limit as x approaches infinity of the rational function **does not exist.**

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22. $\lim_{x \to \infty} \frac{3x^3 + 2}{9x^3 - 2x^2 + 7}$

26. $\lim_{x \to \infty} \left(\frac{1}{2}x - \frac{4}{x^2}\right)$

28. $\lim_{x \to \infty} \frac{x}{\sqrt{x^2 + 1}}$

32. $\lim_{x \to \infty} \frac{x - \cos x}{x}$

56. Graph: $y = \frac{x-3}{x-2}$

66. Graph: $y = \frac{2x}{1-x^2}$

72. Graph: $y = \frac{x}{\sqrt{x^2-4}}$