Math 226 Notes Section 3.6 GRAPHING OVERVIEW

<u>Guidelines for Graphing:</u>

1. Identify what type of graph (general shape) it is, if you can.
2. Find the domain and range.
3. Find the x- and y-intercepts.
4. Find any asymptotes.
5. Locate x-values where f' and f'' are either zero or undefined. Use the results to determine relative extrema and points of inflection.

When graphing rational functions, if the degree of the numerator is 1 higher than the degree of the denominator, then there is a **slant** (or oblique) **asymptote**.

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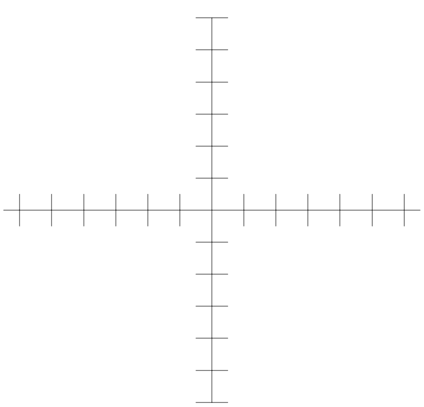
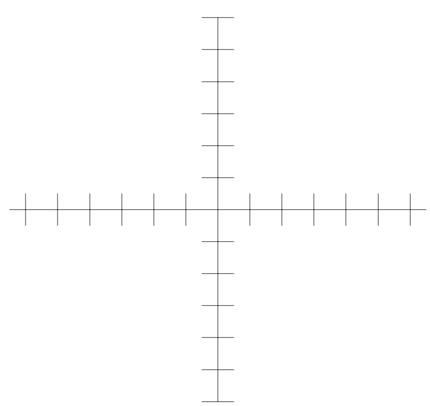
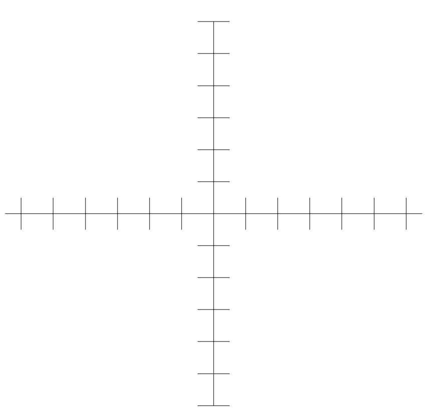
Ex. $y = 3x^4 - 6x^2$



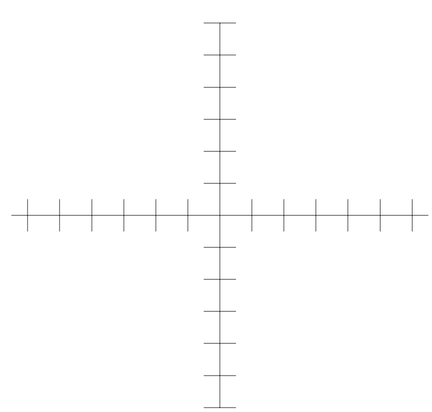
ex. $y = |x^2 - 6x - 5|$



22. $y = x\sqrt{16 - x^2}$



ex. $y = \dfrac{x^3}{x^2 - 1}$



Section 3.7 OPTIMIZATION PROBLEMS

Optimization Steps:

1. Assign symbols for all given quantities and the quantities to be determined. When feasible, make a sketch.
2. Write primary equation for quantity to be maximized or minimized.
3. Plug secondary equations into the primary equation so that the primary equation only has 1 variable.
4. Determine the domain of the primary equation. 9What values make sense?)
5. Take derivative to find the max and min values.

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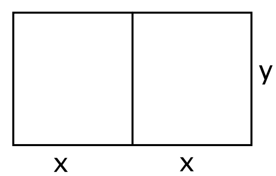
5. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

7. Find two positive integers such that the sum of the first and twice the second is 100 and their product is a maximum.

16. Find the point on $f(x) = (x+1)^2$ that is closest to the point $(5,3)$.

20. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?



25. A right triangle is formed in the first quadrant by the x- and y-axis and a line through the point (1,2).

a. Write the length L of the hypotenuse as a function of x.
b. Use a graphing utility to graphically approximate x such that the length of the hypotenuse is a minimum.
c. Find the equation of the vertices of the triangle such that its area is a minimum.

28. Find ;the dimensions of the largest rectangle that can be inscribed in a semicircle of radius r.

39. A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 12 cubic centimeters. Find the radius of the cylinder that produces the minimum surface area.
40. An industrial tank of the shape described in exercise 35 must have a volume of 3000 cubic feet. The hemispherical ends cost twice as much per square foot of surface area as the sides. Find the dimensions that will minimize cost.

Section 3.8 NEWTON'S METHOD

Newton's Method is a technique for approximating the zeros of a function. It uses tangent lines to approximate the graph of the function near its x-intercepts. It is based on the assumption that f and the tangent line at $(x_1, f(x_1))$ both cross the x-axis at about the same place.

Picture:

<u>NEWTON'S METHOD for Approximating Zeros</u>
Let $f(c) = 0$, where f is differentiable on an open interval containing c.

1. Make an estimate of x_1 that is "close" to c. (A graph can help you.)
2. Determine a new approximation using $x_{n=1} = x_x - \dfrac{f(x_n)}{f'(x_n)}$.
3. If $|x_n - x_{n+1}|$ is within the desired accuracy, let x_{n+1} serve as the final approximation. Otherwise, return to step 2 and calculate a new approximation.

Newton's method doesn't always work. Sometimes it fails to converge to a limit. Newton's method <u>only</u> works if $\left|\dfrac{f(x)f''(x)}{[f'(x)]^2}\right| < 1$

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2. Complete two iterations of Newton's Method for $f(x) = 2x^2 - 3$ using $x_1 = 1$ as the initial guess.

10. Approximate the zero(s) of $f(x) = 1 - 2x^3$ using Newton's Method until two successive approximations differ by less than 0.001.

Section 3.9 DIFFERENTIALS

When the tangent line to the graph of f at the point $(c, f(c))$:

$$y = f(c) + f'(c)(x - c)$$

is used as an approximation the graph of f, the quantity $x - c$ is called the *change in* x, and is denoted by Δx.

When Δx is small, the change in y (denoted by Δy) can be approximated as follows:

$$\Delta y = f(c + \Delta x) - f(c)$$
$$= f'(c)\Delta x$$

Definition of Differentials

Let $y = f(x)$ represent a function that is differentiable in an open interval containing x. The **differential of x** (denoted by **dx**) is any nonzero real number. The **differential of y** (denoted by **dy**) is $dy = f'(x)dx$.

The propagated error is the difference between $f(x + \Delta x)$ and $f(x)$.

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8. Evaluate and compare Δy and **dy** using the given information.

$$y = 1 - 2x^2 \qquad x = 0 \qquad \Delta x = dx = -0.1$$

In exercises 11-20, find the differential **dy** of the given functions.

12. $y = 3x^{2/3}$

16. $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

18. $y = x \sin x$

30. The measurement of the edge of a cube is found to be 12 inches, with a possible error of 0.03 inch. Use differentials to approximate the maximum possible error in computing
a. the volume of the cube.
b. the surface area of the cube.

32. The measurement of the circumference of a circle is found to be 56 centimeters. The possible error in measuring the circumference is 1.2 centimeters.

a. Approximate the percent error in computing the area of the circle.

b. Estimate the maximum allowable percent error in measuring the circumference if the error in computing the area cannot exceed 3%.