

Steps for Solving Related Rates Problems:

- Identify all of the given quantities. Identify all of the quantities to be determined. Make a sketch and label it.
- Write equations involving the variables whose rates of change are either given or are to be determined.
- Differentiate both sides of the equation with respect to time.
- Substitute in all known values. Solve for the required rate of change.

Be sure to differentiate **before** substituting in the known values!

Verbal Statement	Mathematical Model
The velocity of a car after traveling for 1 hour is 50 miles per hour.	$X =$ distance traveled $\frac{dx}{dt} = 50$ when $t=1$
Water is being pumped into a swimming pool at a rate of 10 cubic meters per hour.	$V =$ volume of water in pool $\frac{dV}{dt} = 10m^3/hr$
A gear is revolving at a rate of 25 revolutions per minute.	$\theta =$ angle of revolution $\frac{d\theta}{dt} = 25(2\pi)rad/sec$

Pg. 154 #4 For the equation $x^2 + y^2 = 25$

- Find $\frac{dy}{dt}$ when $x=3$, $y=4$, given $\frac{ds}{dt} = 8$.
- Find $\frac{dx}{dt}$ when $x=4$, $y=3$, given $\frac{dy}{dt} = -2$.

14. Find the rate of change of the distance between the origin and a moving point on the graph of

$$y = \sin x \text{ if } \frac{dx}{dt} = 2 \text{ centimeters per second.}$$

18. The radius r of a sphere is increasing at a rate of 2 inches per minute.

- Find the rate of change of the volume when $r=6$ inches and $r=24$ inches.
- Explain why the rate of change of the volume of the sphere is not constant even though $\frac{dr}{dt}$ is constant.

24. A conical tank (with vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.

26. A trough is 12 feet long and 3 feet across the top. Its ends are isosceles triangles with altitudes of 3 feet. If water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when the water is 1 foot deep?

28. A construction worker pulls a 5-meter plank up the side of a building under construction by means of a rope tied to one end of a plank. Assume the opposite end of the plank follows a path perpendicular to the wall of the building and the worker pulls the rope at a rate of 0.15 meters per second. How fast is the end of the plank sliding along the ground when it is 2.5 meters from the wall of the building?

32. An airplane is flying at an altitude of 6 miles and passes directly over a radar antenna. When the plane is 10 miles away ($s=10$), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the plane?

35. A man 6 feet tall walks at a rate of 5 feet per second away from a light that is 15 feet above the ground. When he is 10 feet from the base of the light.

- a. at what rate is the tip of his shadow moving?
- b. at what rate is the length of his shadow changing?

36. Repeat the exercise at the left for a man 6 feet tall walking at a rate of 5 feet per second **toward** a light that is 20 feet above the ground.

42. Cars on a certain roadway travel on a circular arc of radius r . In order not to rely on friction alone to overcome the centrifugal force, the road is banked at an angle of magnitude θ from the horizontal. The banking angle must satisfy the equation $rg \tan \theta = v^2$, where v is the velocity of the cars and $g=32$ feet per second per second is the acceleration due to gravity. Find the relationship between the related rates $\frac{dv}{dt}$ and $\frac{d\theta}{dt}$.

44. A fish is reeled in at a rate of 1 foot per second from a point 15 feet above the water. At what rate is the angle between the line and the water changing when there are 25 feet of line out?

46. A patrol car is parked 50 feet from a long warehouse. The revolving light on top of the car turns at a rate of 30 revolutions per minute. How fast is the light beam moving along the wall the beam makes an angle of (a) $\theta = 30^\circ$, (b) $\theta = 60^\circ$, and (c) $\theta = 70^\circ$ with the line perpendicular from the light to the wall?

48. An airplane is flying in still air with an airspeed of 240 miles per hour. If it is climbing at an angle of 22° , find the rate at which it is gaining altitude.

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34. A ball is dropped from a height of 100 feet. One second later, another ball is dropped from a height of 75 feet. Which ball hits the ground first?

111. The cross section of a 5-meter trough is an isosceles trapezoid with a 2-meter lower base, a 3-meter upper base, and an altitude of 2 meters. Water is running into the trough at a rate of 1 cubic meter per minute. How fast is the water level rising when the water is 1 meter deep?

In exercises 101-106, use implicit differentiation to find $\frac{dy}{dx}$.

102. $x^2 + 9y^2 - 4x + 3y = 0$

104. $y^2 = (x - y)(x^2 + y)$

106. $\cos(x + y) = x$