12.1 <u>One-Way Analysis of Variance</u>

<u>ANOVA</u> - analysis of variance - used to compare the means of <u>several</u> populations.

<u>Assumptions for One-Way ANOVA</u>:
1. Independent samples are taken using a randomized design.
2. For each population, the variable under consideration is normally distributed.
3. The standard deviations of the variable under consideration are the same for all the populations.

example: Consider comparing mean sales within 4 regions of the U.S. This meets all 3 assumptions above.

Ho: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (the mean sales are all equal)

H_1: all the means are not equal

<u>process:</u> Take random independent samples from each region. Compare the means from all 4 samples. Then make a decision about Ho.

notation:

k= # of populations sampled

n= total umber of data entries (from all populations)

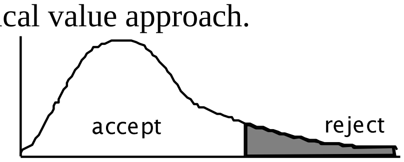
n_1, n_2, n_3 ... = total number of data entries in <u>each</u> population.

<u>One-Way ANOVA Hypothesis Tests</u>

step 1: State Ho and H_1.

step 2: Decide on significance level.

step 3: The critical value is F_α with df=(k-1, n-k) if you use a critical value approach.



step 4: Enter values into calculator to construct a one-way ANOVA Table. Identify P-value.

Source	df	SS	MS	F-statistic
Factor	$k-1$	SSTR	MSTR	$F = \frac{MSTR}{MSE}$
Error	$n-k$	SSE	MSE	
Total	$n-1$	SST		

Step 5: Accept/ Reject Ho.

Step 6: State conclusion in words.

example: Manufacturers of golf balls seem to always claim that their ball goes farthest. 20 golf pros are randomly selected to test 5 brands with 4 golf pros per brand. Here are the results of each drive (in yards):

Brand 1	Brand 2	Brand 3	Brand 4	Brand 5
286	279	270	284	281
276	277	262	271	293
281	284	277	269	276
274	288	280	275	292

Do the data provide sufficient evidence to conclude that a difference in mean driving distances exists? Use α=0.05.

Source	df	SS	MS	F-statistic
Factor				
Error				
Total				

Note that the ANOVA test cannot tell you which brand drives the farthest. It can only tell you whether a difference exists in the mean driving distances.

example: A chain of convenience stores wanted to test three different advertising policies:

- Policy 1: No advertising.
- Policy 2: Advertise in neighborhoods with circulars.
- Policy 3: Use circulars and advertise in neighborhoods.

Eighteen stores were randomly selected and divided randomly into three groups of six stores. Each group used one of the three policies. Following the implementation of the policies, sales figures were obtained for each of the stores during a 1-month period. The figures are displayed, in thousands of dollars, in the following table:

POLICY 1	POLICY 2	POLICY 3
22	21	29
20	25	24
26	25	31
21	20	32
24	22	26
22	26	27

At the 1% significance level, do the data provide evidence of a difference in mean monthly sales among the three policies?

Source	**df**	**SS**	**MS**	**F-statistic**
Factor				
Error				
Total				

13.1 <u>Nonparametric Statistics</u>

Nonparametric statistics (or distribution-free statistics) are used when the population from which the samples are selected is not normally distributed. Nonparametric statistics can also be used to test hypotheses that do not involve specific population parameters such as μ, σ or p.

<u>Advantages and Disadvantages of Nonparametric Statistics Over Parametric Methods:</u>

Advantages:

- They can be used to test population parameters when the variable is not normally distributed.
- They can be used when the data are nominal or ordinal.
- They can be used to test hypotheses that do not involved population parameters.
- In most cases, the computations are easier than those for parametric counterparts.

Disadvantages:

- They are less sensitive than their parametric counterparts when the assumptions of the parametric methods are met. Therefore, larger differences are needed before the null hypothesis can be rejected.
- They tend to use less information than the parametric tests. For example, the sign test requires the researcher to determine only whether the data values are above or below the median, not how much above or below the median each value is.
- They are less efficient than their parametric counterparts when the assumptions of the parametric method are met. That is, larger sample sizes are needed to overcome the loss of information. For example, the nonparametric sign test is about 60% as effective as its parametric counterpart, the *z*-test. Thus, a sample size of 100 is needed for the use of the sign test, compared with a sample size of 60 for use of the *z*-test to obtain the same result.

Many nonparametric tests involve the ranking of data, that is, the positioning of a data value in an array according to some rating scale.

13.2 <u>The Sign Test for Single and Paired Samples</u>

The simplest nonparametric test, the **sign test** for single samples, is used to test the value of a median for a specific sample. When using the sign test, the researcher hypothesizes the specific value for the median of a population; then he or she selects a sample of data and compares each value with the conjectured median. If the data value is above the conjectured median, it is assigned a plus sign. If it is below the conjectured median, it is assigned a minus sign. And if it is exactly the same as the conjectured median, it is assigned a 0. Then the number of plus and minus signs are compared. If the null hypothesis is true, the number of plus signs should be approximately equal to the number of minus signs. If the null hypothesis is not true, there will be a disproportionate number of plus or minus signs.

<u>General Procedure for the Sign Test</u> (Critical Value Approach):
Assumptions: Independent samples and same-shaped populations

1. State Ho (median = value) and H_1 (median $\neq$ value)
2. Determine significance level α.
3. Find the critical value.
   a. For the single-sample test, compare each value of the data with the median. If the value is greater than the median, replace the value with a plus sign. If it is less than the median, replace it with a minus sign. If it is equal to the median, replace it with a 0.
   b. For the paired-sample sign test, subtract the after values from the before values, and indicate the difference with a positive or negative sign or 0.
   c. Refer to table for the critical value. The value of n is equal to the number of plus and minus signs you have created.
4. Compute the test statistic. Count the number of plus and minus signs obtained above and use the smaller value as the test statistic.
5. Make the decision. If the test statistic is less than or equal to the critical value, reject H_0.
6. State your conclusion in words.

Critical Values for the Sign Test (Single and Paired Samples)

Reject the null hypothesis if the smaller of + or – signs is less than or equal to the value in this table.

	One-tailed, $\alpha = 0.005$	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
n	Two-tailed, $\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$
8	0	0	0	1
9	0	0	1	1
10	0	0	1	1
11	0	1	1	2
12	1	1	2	2
13	1	1	2	3
14	1	2	3	3
15	2	2	3	3
16	2	2	3	4
17	2	3	4	4
18	3	3	4	5
19	3	4	4	5
20	3	4	5	5
21	4	4	5	6
22	4	5	5	6
23	4	5	6	7
24	5	5	6	7
25	5	6	6	7

When the sample size is 26 or more, the normal approximation can be used to find the test statistic using the formula below. The critical value(s) would then use the normal curve (Inverse Norm).

Formula for the z Test Value in the Sign Test When $n \geq 26$

$$z = \frac{(X + 0.5) - (n/2)}{\sqrt{n}/2}$$

where

X = smaller number of + or − signs
n = sample size

Example:

6. A meteorologist suggests that the median temperature for the month of July in Jacksonville, Florida, is 81°F. The sample here shows the temperatures taken at noon in Jacksonville during 20 days in July. At $\alpha = 0.01$, is there enough evidence to reject the meteorologist's claim?

81	83	87	92	91
78	73	81	93	96
79	80	84	86	82
85	77	72	73	80

Example:

8. A government economist estimates that the median cost per pound of beef is \$5.00. A sample of 22 livestock buyers shows the following costs (in dollars) per pound of beef. Is there enough evidence to reject the economist's hypothesis at $\alpha = 0.10$?

5.35	5.16	4.97	4.83	5.05	5.19
4.78	4.93	4.86	5.00	4.63	5.06
5.19	5.00	5.05	5.10	5.16	5.25
5.16	5.42	5.13	5.27		

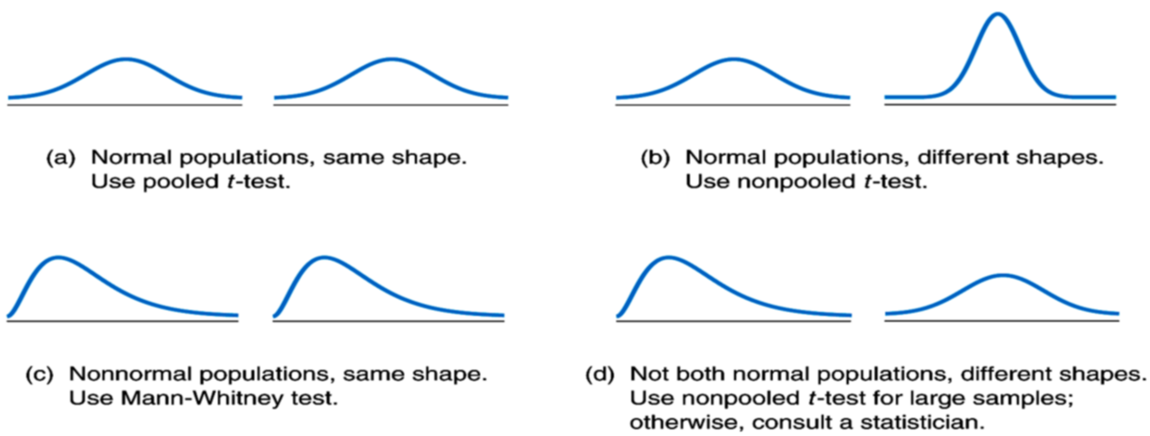
Example: pg. 679 #17
Is there a difference in weekend movie attendance based on the evening in question? Eight small-town theaters were surveyed to see how many movie patrons were in attendance on Saturday evening and Sunday evening. Is there sufficient evidence to reject the claim that there is no difference in movie attendance for Saturday and Sunday evenings? Use a 10% significance level.

Theater	A	B	C	D	E	F	G	H
Saturday	210	100	150	50	195	125	120	204
Sunday	165	42	92	60	172	100	108	136

13.3 <u>The Mann-Whitney Test (a.k.a. the Wilcoxon Rank Sum Test)</u>

Another procedure for performing a hypothesis test based on independent samples to compare the mean of two populations is the Mann-Whitney test. This nonparametric test applies when the two distributions of the variable under consideration have the same shape, but it does not require that they be normal or have any other specific shape.

Appropriate procedure for comparing two population means using independent samples



Assumptions: Independent samples and same-shaped populations

1. State Ho (use the word "same") and H_1 (use the word "different").
2. Determine significance level α. (This will always be given to you.)
3. The critical value(s) are found using InvNorm (just as you originally learned for hypothesis tests).
4. Construct a work table of the following form:

Sample from Population 1	Overall rank	Sample from Population 2	Overall rank
.	.	.	.
.	.	.	.
.	.	.	.

   Note: When there are ties in the sample data, ranks are assigned in the same way as the Wilcoxon signed rank test. Namely, if two or more observations are tied, each is assigned the mean of the ranks they would have had if there were no ties.

5. Compute the value of the test statistic:
   a. Calculate R=sum of the ranks for sample data from the population with the smallest sample size. (If both are the same size, either one can be used.)
   b. Calculate $\mu_R = \frac{n_1(n_1+n_2+1)}{2}$ and $\sigma_R = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}$ (where n_1 and n_2 are each greater than or equal to 10).
   c. Find the test statistic: $z = \frac{R-\mu_R}{\sigma_R}$.
6. Decide whether to accept or reject Ho. Is z in the acceptance or rejection region?
7. State your conclusion in words.

When deciding upon whether to use the pooled-t test or the Mann-Whitney test, use the following guideline: *If you are reasonably sure that the two distributions are normal (perhaps by examining a normal probability plot), use the pooled-t test; otherwise use the Mann-Whitney.*

Example: Independent random samples of male and female workers gave the following data on weekly earnings, in dollars.

Men	826	1790	477	307	2523	288	317	718		
Women	1982	498	414	278	1349	2097	279	262	1085	316

At the 5% significance level, do the data provide sufficient evidence to conclude that the median weekly earnings of male full-time wage and salary workers exceeds the median weekly earnings of female full-time wage and salary workers?

Sample from Population 1	Overall rank	Sample from Population 2	Overall rank
826		1982	
1790		498	
477		414	
307		278	
2523		1349	
288		2097	
317		279	
718		262	
		1085	
		316	

Example: Supervisors were asked to rate the productivity of employees on their jobs. A researcher wishes to see whether married men receive higher ratings than single men. A rating scale of 1 to 50 yielded the data shown below. At a 1% significance level, is there evidence to support this claim?

Single Men	Overall rank	Married Men	Overall rank
48		44	
46		35	
42		41	
50		37	
38		42	
36		43	
40		29	
31		31	
28		37	
24		32	
49		36	
34			

13.4 The Paired Wilcoxon Signed-Rank Test

The Wilcoxon signed-rank test is based on the assumption that the variable under consideration has a **symmetric** distribution, but does not require normality or a specific shape. This will be the main distinction in determining whether to apply a t-test procedure or use the Wilcoxon signed-rank test.

General Procedure for the Paired Wilcoxon Signed-Rank Test (Critical Value Approach):
Assumption: Symmetric Distribution

1. State H_0 and H_1.
2. Determine significance level α □.
3. The critical values are found using the table provided.
4. Construct a work table to help calculate the test statistic:

Before, X_B	After, X_A	Difference $D=X_B-X_A$	$\lvert D\rvert$	Rank of $\lvert D\rvert$	Signed Rank R
.		.	.	.	.
.		.	.	.	.
.		.	.	.	.

5. Compute the value of the test statistic:
   Find the sum of the positive and negative ranks separately. Select the smaller of the absolute value of the sums as the test statistic W.
6. Decide whether to accept or reject Ho. Is the W in the rejection (less than or equal to the critical value) or acceptance region (greater than the critical value)?
7. State your conclusion in words.

The following points may be needed when performing a Wilcoxon signed-rank test:

- If an observation equals μ_0 (the value for the mean in Ho), that observation should be removed and the sample size reduced by 1.
- If two or more absolute differences are tied, each should be assigned the mean of the ranks they would have had if there were no ties. For example, if two absolute value differences are tied for second place, each should be assigned $\frac{2+3}{2}=2.5$, and then rank 4 should be assigned to the next largest absolute difference, which really is fourth. If three absolute differences are tied for fifth place, each should be assigned rank $\frac{5+6+7}{3}=6$, and rank 8 should be assigned to the next largest absolute difference.

Critical Values for the Wilcoxon Signed-Rank Test
Reject the null hypothesis if the test statistic is less than or equal to the value given in this table.

n	One-tailed, $\alpha=0.05$	$\alpha=0.025$	$\alpha=0.01$	$\alpha=0.005$
	Two-tailed, $\alpha=0.10$	$\alpha=0.05$	$\alpha=0.02$	$\alpha=0.01$
5	1			
6	2	1		
7	4	2	0	
8	6	4	2	0
9	8	6	3	2
10	11	8	5	3
11	14	11	7	5
12	17	14	10	7
13	21	17	13	10
14	26	21	16	13
15	30	25	20	16
16	36	30	24	19
17	41	35	28	23
18	47	40	33	28
19	54	46	38	32
20	60	52	43	37
21	68	59	49	43
22	75	66	56	49
23	83	73	62	55
24	92	81	69	61
25	101	90	77	68
26	110	98	85	76
27	120	107	93	84
28	130	117	102	92
29	141	127	111	100
30	152	137	120	109

Because the mean and median of a symmetric distribution are identical, a Wilcoxon signed-rank test can be used to on hypothesis tests for a population median, η, as well as for a population mean μ.

Example: Eight couples are given a questionnaire designed to measure marital compatibility. After completing a worksheet, they are given a second questionnaire to see whether there is a change in their attitudes toward each other. At a 10% significance level, is there any difference in the scores of the couples?

Before, X_B	After, X_A	Difference $D=X_B-X_A$	$\lvert D\rvert$	Rank of $\lvert D\rvert$	Signed Rank R
43	48				
52	59				
37	36				
29	29				
51	60				
62	68				
57	59				
61	72				

Example: pg. 690 #10 In a corporation, male and female workers were matched according to years of experience working for the company. Their salaries were then compared. The data (in thousands of dollars) are shown in the table. At the 10% significance level, is there a difference in the salaries of the males and females?

Males, X_B	Females, X_A	Difference $D=X_B-X_A$	$\lvert D \rvert$	Rank of $\lvert D \rvert$	Signed Rank R
18	16				
43	38				
32	35				
27	29				
15	15				
45	46				
21	25				
22	28				

13.5 The Kruskal-Wallis Test

In this section we learn how to perform a Kruskal-Wallis test, a nonparametric alternative to the one-way ANOVA procedure. The Kruskal-Wallis test applies when the distributions (one for each population) of the variable under consideration have the same shape; it does not require that the distributions be normal or have any other specific shape.

Like the Mann-Whitney test, the Kruskal-Wallis test is based on ranks. When ties occur, ranks are assigned in the same way as in the Mann-Whitney test. If two or more observations are tied, each tie is assigned to the mean of the ranks they would have had if there were no ties.

General Procedure for the Kruskal-Wallis Test (Critical Value Approach):

Assumptions: 1. Independent samples
2. same-shaped populations
3. All sample sizes are 5 or greater

1. State Ho and H_1 (same as with ANOVA tests)
2. Determine significance level α?. (This will always be given to you.)
3. Construct a work table of the following form:

Sample from Population 1	Overall rank	Sample from Population 2	Overall rank	…	Sample from Population k	Overall rank
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.

4. Compute the value of the test statistic:
$H = \frac{SSTR \cdot (n-1)}{SST}$, where SSTR and SST can be found by running an ANOVA test on the columns containing the ranks. (On the TI-83, SSTR is the "Factor SS" and SST can be found by adding the "Factor SS" and the "Error SS" together.)
5. The critical value is χ^2_α with $df = k-1$, where k is the number of populations. Use an INVERSE CHI procedure.
6. Decide whether to accept (test statistic is less than critical value) or reject H_0.
7. State your conclusion in words.

Example: Independent random samples of surveys on consumer expenditures for various types of entertainment yielded the following data, in dollars, on last year's expenditures for three entertainment categories.

At the 5% significance level, do the data provide sufficient evidence that a difference exists in last year's mean expenditures among the three entertainment categories? Perform a Kruskal-Wallis test.

Fees and Admissions	**TV, radio and Sound equipment**	**Other equipment and services**
173	100	0
112	1748	251
22	396	1293
495	0	31
111	470	75
1203	0	1024
609	562	1629
300		102
		1238

Sample from Population 1	Overall rank	Sample from Population 2	Overall rank	Sample from Population 3	Overall rank
173		100		0	
112		1748		251	
22		396		1293	
495		0		31	
111		470		75	
1203		0		1024	
609		562		1629	
300				102	
				1238	

Example: Independent random samples of new car buyers yielded the following data on age of purchaser, in years, by origin of car purchased.

Do the data provide sufficient evidence to conclude that a difference exists in the median ages of buyers of new domestic, Asian, and European cars? Perform a Kruskal-Wallis test using $\alpha = 0.05$.

Domestic	**Asian**	**European**
41	78	72
42	42	42
51	51	58
47	45	39
33	21	67
83	24	39
35	21	45
69	39	27
50	45	33
60	30	55

Sample from Population 1	**Overall rank**	**Sample from Population 2**	**Overall rank**	**Sample from Population 3**	**Overall rank**
41		78		72	
42		42		42	
51		51		58	
47		45		39	
33		21		67	
83		24		39	
35		21		45	
69		39		27	
50		45		33	
60		30		55	

➢ Math 120 – Final Exam Notes

Calculate a confidence (or prediction) interval
Calculate binomial probabilities (binompdf)
Calculate conditional probabilities
Calculate expected values
Calculate measures of center or spread
Calculate probabilities
Describe a Type 1 or Type 2 error in context
Find outliers (mathematically)
Find test statistics for hypothesis tests
Find the area under normal curves
Find the equation of a regression line
Find z-scores (InverseNorm)
Independent versus dependent events
Methods of sampling
Normal curves – different shapes (mu, sigma)
Probability – expected values in the long run
Setting up hypothesis tests, null and alternate hypotheses
Shapes of graphs (symmetry, skewness)
Sketch and shade area under a normal curve
Statistics versus parameters
Use a regression equation to make a prediction
When to use t versus z
Wording of conclusions of hypothesis tests
Wording of confidence intervals

<u>Murphy's Laws and Mathematics</u>

Murphy's law and its corollaries are familiar to everyone who studies mathematics.

- Murphy's Law: If anything can go wrong, it will.
- Corollary 1: At the worst possible time
- Corollary 2: Causing the most damage

Here are some ways in which Murphy's law applies to mathematics:

- The harder you study, the farther behind you get.
- Every problem is harder than it looks and takes longer than you expected.
- When you solve a problem, it always helps to know the answer.
- Any expression can be made equal to any other expression if you juggle it enough.
- Knowing mathematics and teaching mathematics are not equivalent.
- Teaching ability is inversely proportional to the number of papers published.
- Proofs don't convince anybody of anything.
- An ounce of example is worth a pound of theory.
- What is "obvious" to everyone else won't be "obvious" to you.
- Notes you understood perfectly in class transform themselves into hieroglyphics at home.
- Textbooks are written for those who already know the subject.
- Any simple idea will be expressed in incomprehensible terms.
- The answers you need aren't in the back of the book.
- No matter how much you study for exams, it will never be enough.
- The problems you can work are never put on the exam.
- The problems you are certain won't be on the test will be.
- The answer to the problem you couldn't work on the exam will become obvious after you hand in your paper.