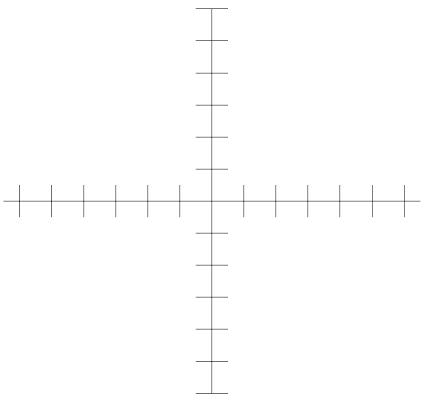
Math 226 Notes Section 7.1 THE AREA BETWEEN TWO CURVES

We have skipped chapter 5 which deals with logs. We will need to be careful to skip those exercises that require integration and differentiation of logs. You can still read the examples in the text, however to learn the processes involved.

To find the area between two curves, evaluate the following integral: $\int_a^b [f(x) - g(x)]dx$.

If the representative rectangles in your problem are vertical, differentiate with respect to **x**, using **dx**.) If the representative rectangles are horizontal, differentiate with respect to **y** and use **dy**.

Sometimes you will have to solve for the intersection of the two curves to find the upper and lower limits of integration.



$$A = \int_{x_1}^{x_2} (\textit{top curve} - \textit{lower curve})\, dx$$

$$A = \int_{y_1}^{y_2} (\textit{right curve} - \textit{left curve})\, dy$$

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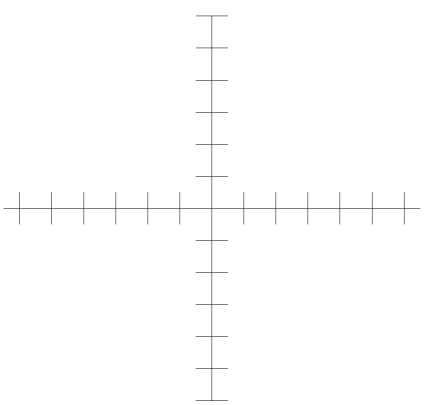
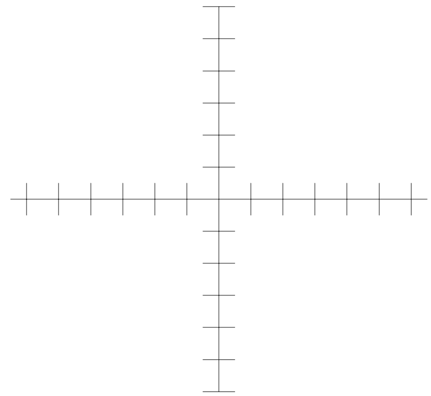
2. Write a definite integral to represent the following shaded region:

$$f(x) = x^2 + 2x + 1$$
$$g(x) = 2x + 5$$



8. Sketch the graph of each function and shade the area represented by the integral.

$$\int_{-1}^{1} \left[\left(1 - x^2\right) - \left(x^2 - 1\right)\right] dx$$



22. Sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

$$f(x) = -x^2 + 4x + 2;\ g(x) = x + 2$$

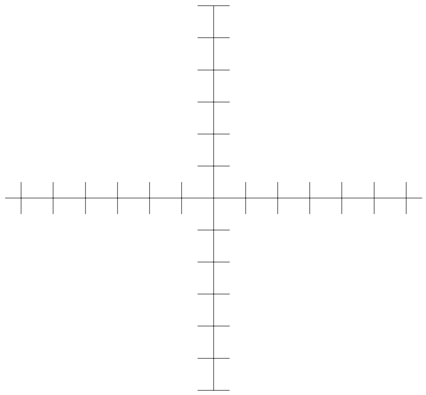
Ex. Sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

$$f(x) = \sqrt[3]{x}; \; g(x) = x$$

34. Use a graphing utility to find the boundaries of integration and then integrate.

$$f(x) = x^3 - 2x + 1; \; g(x) = -2x; \; x = 1$$

61. Use integration to find the area of the triangle having the vertices $(2,-3),(4,6),(6,1)$.



66. See book

Section 7.2 VOLUME: THE DISC METHOD

In this section, we find the volume of a solid by rotating a curve about a line:

Volume of disc = (area of disc)(width of disc)

$$= \pi R^2 w$$

$$\Delta V = \pi R^2 \Delta x$$

Key words: Solid of revolution, axis of revolution

<u>The disc method:</u>

Horizontal axis of revolution: $V = \pi \int_a^b [R(x)]^2 dx$

Vertical axis of revolution: $V = \pi \int_c^d [R(y)]^2 dy$

<u>The washer method:</u>

$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2\right) dx$, where ***R*** is the outer radius and ***r*** is the inner radius.

If you are going to form a solid by rotating a shape of known area about an axis of revolution, use one of the following formulas:

$$V = \int_a^b A(x)dx \text{ or } V = \int_c^d A(y)dy$$

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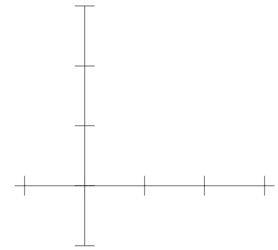
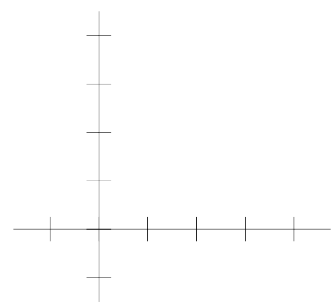
4. $y = \sqrt{9 - x^2}$

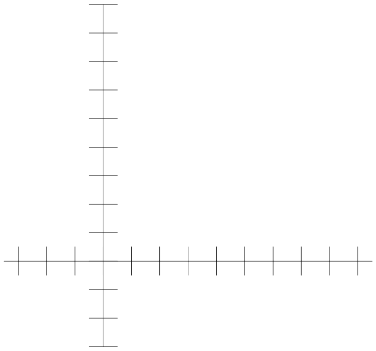


10. $x = -y^2 + 4y$

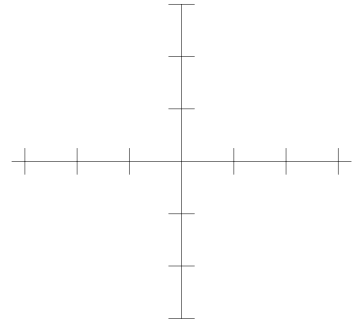


12. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = 2x^2$, $y = 0$, and $x = 2$ about the given lines.

a) the y-axis
b) the x-axis
c) the line **y=8**
d) the line **x=2**



24. Find the volume of the solid generated by revolving the region bounded by $y = 0$ and $y = x\sqrt{4 - x^2}$ about the x-axis.



52. The region bounded by $y = \sqrt{x}$, $y = 0$, $x = 0$, and $x = 4$ is revolved about the x-axis.

a. Find the value of x in the interval $[0,4]$ that divides the solid into two parts of equal volume.
b. Find the value of x in the interval $[0,4]$ that divides the solid into three parts of equal volume.

61. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross section taken perpendicular to the x-axis.

a. Squares

b. rectangles of height 1

Section 7.3 THE SHELL METHOD

<u>Volume of a Shell:</u>

$$V = \pi\left(p + \frac{w}{2}\right)^2 h - \pi\left(p - \frac{w}{2}\right)^2 h$$

$$= \pi\left(p^2 + pw + \frac{w^2}{4}\right)h - \pi h\left(p2 - pw + \frac{w^2}{4}\right)$$

$$= \pi h\left(p^2 + pw + \frac{w^2}{4} - p^2 + pw - \frac{w^2}{4}\right)$$

$$= \pi h\,(2pw)$$

$$= 2\,\pi\,p\,h\,w$$

$$V = 2\,\pi\,\text{(average radius) (height) (thickness)}$$

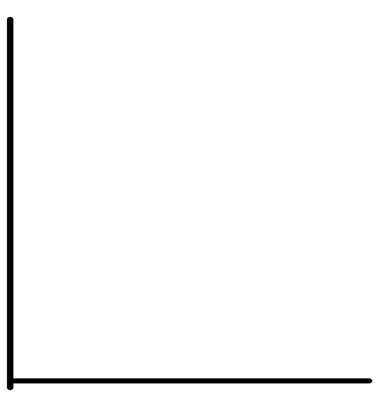
When there's a <u>horizontal</u> axis of revolution,
use $V = 2\,\pi \int_c^d p(y)h(y)dy$

When there's a <u>vertical</u> axis of revolution,
use $V = 2\,\pi \int_a^b p(x)h(x)dx$

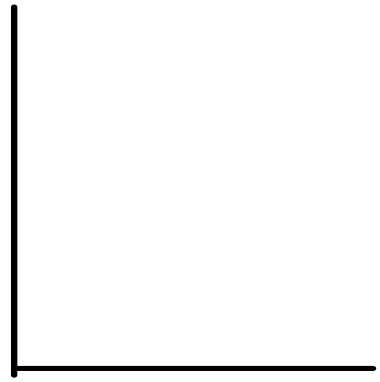
<u>Disc versus Shell:</u>
For the disc method, the representative rectangles are <u>perpendicular</u> to the axis of rotation, whereas for the shell method, the representative rectangles are <u>parallel</u> to the axis of rotation.

- or **p(y)** is the distance to the axis of rotation
- or **h(y)** is the height if the representative rectangle.

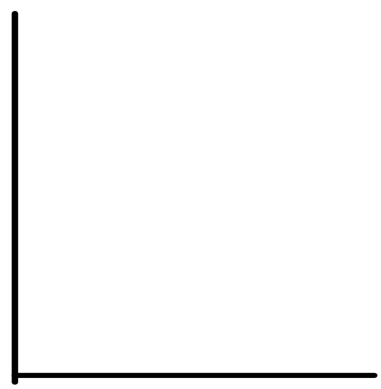
$$V = \pi \int_c^d \left(R^2 - r^2\right)dy$$



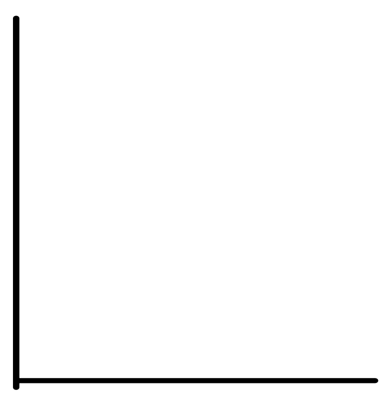
$$V = \pi \int_a^b \left(R^2 - r^2\right)dx$$



$$V = 2\pi \int_a^b ph\,dx$$



$$V = 2\pi \int_c^d ph\,dy$$



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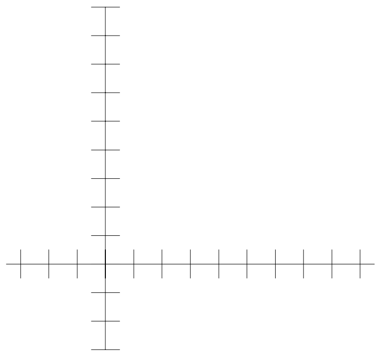
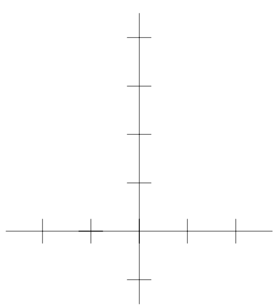
In problems 1-12, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

4. $y = x^2 + 4$



8. $y = 4 - x^2$
$y = 0$

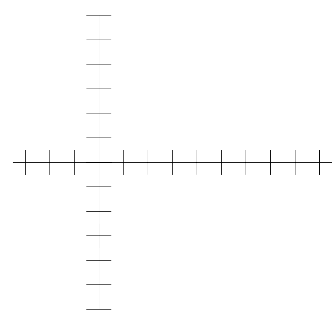


12. $y = \begin{cases} \dfrac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}, \quad y = 0, \quad x = 0, \quad x = \pi$

16. Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

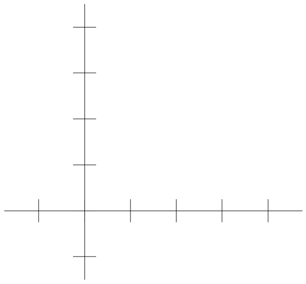
$x + y^2 = 16$

$x = 0$



24. Use the shell method to find the volume of the solid generated by revolving the plane region about the line **x=6**.

$y = \sqrt{x}$
$y = 0$
$x = 4$



28. Use the disc or the shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated line.

$$y = \frac{10}{x^2},\ y=0,\ x=1,\ x=4$$

a. the x-axis
b. the y-axis
c. the line **y=1**.

42. A solid is generated by revolving the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$ about the y-axis. A hole, centered along the axis of revolution, is drilled through the solid so that one-third of the volume is removed. Find the diameter of the hole.