A curve is rectifiable if it has a finite arc length. It is sufficient that f' be continuous on $[a,b]$ in order for f to be rectifiable between $(a,f(a))$ and $(b,f(b))$. We will consider graphs of smooth curves that are continuously differentiable.

Arc Length

The arc length of f between a and b is given by
$s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$.

The arc length of g between c and d is given by
$s = \int_c^d \sqrt{1+[g'(y)]^2}\,dy$.

Note that s is being used here to denote distance. Some prefer to use the variable L.

If the graph of a continuous function is revolved about a line, the result is a surface of revolution.

Note: $S = 2\pi r l$, the lateral surface area of a frustrum, where $r = \frac{1}{2}(r_1 + r_2)$.

Area of a Surface of Revolution

In these formulas, $r(x)$ is the radius of revolution.

$$S = 2\pi \int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$S = 2\pi \int_c^d r(y)\sqrt{1+[g'(y)]^2}\,dy$$

problems - pages 483-484

2. Find the distance between $(1,2)$ and $(7,10)$ by using both the distance formula and integration.

6. Find the arc length of the graph of $y = \frac{x^4}{8} + \frac{1}{4x^2}$ over the interval $[1,2]$.

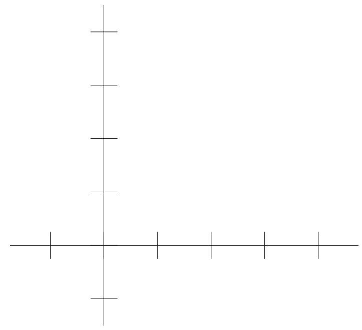
In exercises 15-24, a) graph the function, highlighting the part indicated by the given interval, b) find the definite integral that represents the arc length of the curve over the indicated interval and observe that the integral cannot be evaluated with the techniques studied thus far, and c) use a graphing calculator to find the approximate arc length.

16. $y = x^2 + x - 2$ over the interval $-2 \le x \le 1$.

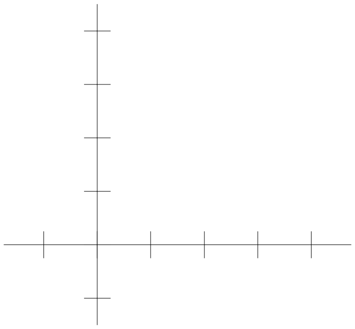
20. $y = \cos x$ over the interval $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$.

24. $x = \sqrt{36 - y^2}$ over the interval $0 \le y \le 3$.

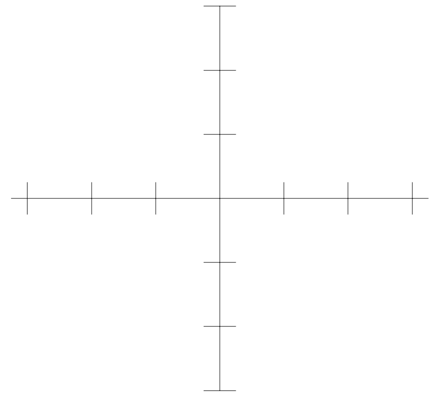
40. Set up and evaluate the definite integral for the area of the surface generated by revolving $y = \sqrt{x}$ about the x-axis over the interval $[4, 9]$.



44. Set up and evaluate the definite integral for the area of the surface generated by revolving $y = 9 - x^2$ about the y-axis over the interval $[0,3]$.



53. Find the surface area of the zone of the sphere formed by revolving the graph of $y = \sqrt{9 - x^2}$, $0 \le x \le 2$, about the y-axis.



Section 7.5 WORK

problems - page 494

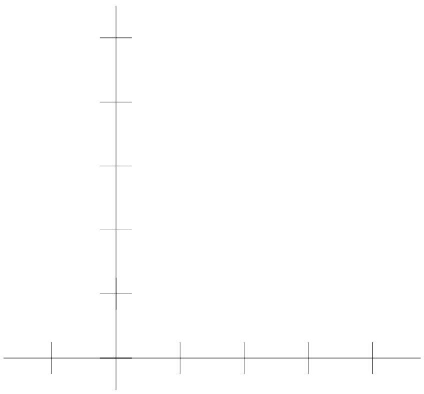
24. A cylindrical water tank 4 meters high with a radius of 2 meters is situated on a tower such that the bottom of the tank is 10 meters above the level of a stream. How much work is done in filling the tank half full of water through a hole in the bottom, using water from the stream?

Water weighs 1000 kg per cubic meter

26. An open tank has the shape of a right circular cone. The tank is 8 feet across the top and 6 feet high. If water is pumped in through the bottom of the tank, how much work is done to fill the tank
a. to a depth of 2 feet?
b. from a depth of 4 feet to a depth of 6 feet?

Water weighs **62.4** lbs per cubic foot

28. The fuel tank on a truck has trapezoidal cross sections with dimensions shown in the figure. Assume that the engine is approximately 3 feet above the top of the fuel tank and that diesel fuel weighs approximately 53.1 pounds per cubic foot. Find the work done by the fuel pump in raising a full tank to the level of the engine.



30. The top of a cylindrical storage tank for gasoline at a service station is 4 feet below ground level. The axis of the tank is horizontal and its diameter and length are 5 feet and 12 feet. Find the work done in pumping the entire contents of the fuel tank to a height of 3 feet above ground level.

Section 7.6 MOMENTS, CENTERS OF MASS, CENTROIDS

problems - pages 604-506

In #1-4, find the center of mass of the point masses lying on the x-axis.

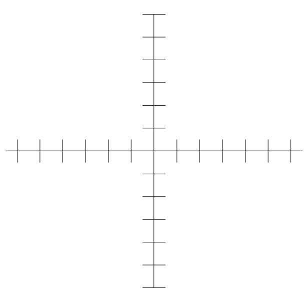
Formula:

$$\bar{x} = \frac{M_0}{m}, \text{ where } \begin{cases} M_0 = m_1x_1 + m_2x_2 + \ldots + m_nx_n \\ m = m_1 + m_2 + \ldots + m_n \end{cases}$$

2. $m_1 = 7,\ m_2 = 4,\ m_3 = 3,\ m_4 = 8$
$x_1 = -3,\ x_2 = -2,\ x_3 = 5,\ x_4 = 6$

4. $m_1 = 12,\ m_2 = 1,\ m_3 = 6,\ m_4 = 3,\ m_5 = 11$
$x_1 = -6,\ x_2 = -4,\ x_3 = -2,\ x_4 = 0,\ x_5 = 8$

10. Find the center of mass of the given system of point masses:
$m_1 = 10$ at $(1,-1)$
$m_1 = 2$ at $(5,5)$
$m_1 = 5$ at $(-4,0)$



Formulas:

$$\bar{x} = \frac{M_y}{m} = \frac{m_1x_1 + m_2x_2 + \ldots + m_nx_n}{m_1 + m_2 + \ldots + m_n}$$

$$\bar{y} = \frac{M_x}{m} = \frac{m_1y_1 + m_2y_2 + \ldots + m_ny_n}{m_1 + m_2 + \ldots + m_n}$$

In exercises 13-24, find M_x, M_y, and $(\bar{x}, \bar{y})$ for the laminas (planar regions) of uniform density ρ bounded by the graphs of the equations.

Formulas:

$$m = \rho\int_a^b [f(x) - g(x)]dx$$

$$M_x = \rho\int_a^b \left[\frac{f(x) + g(x)}{2}\right](f(x) - g(x))dx$$

$$M_y = \rho\int_a^b x(f(x) - g(x))dx$$

$$\bar{y} = \frac{M_x}{m}; \quad \bar{x} = \frac{M_y}{m}$$

ex. $y = x^2,\ y = 0,\ x = 4$

ex. $y = \sqrt{3x} + 1, \quad y = x + 1$

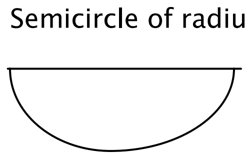
37. Find the centroid of the region bounded by the graphs of $y = 0$ and $y = \frac{b}{a}\sqrt{a^2 - x^2}$.

44. Indicate an appropriate coordinate system and find the coordinates of the center of pass of the planar lamina:

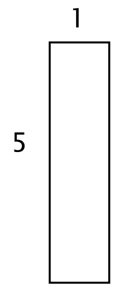
Section 7.7 FLUID PRESSURE

problems - page 511

8. Find the fluid force on the vertical side of the tank, where the dimensions are given in feet. Assume the tank is full of water. The weight-density of water is **62.4** pounds per cubic foot.



14. Find the fluid force on the vertical plate submerged in water, where the dimensions are given in meters and the density of water is 1000 kg per cubic meter.



18. The triangle shown is the vertical side of a form for poured concrete that weighs 140.7 pounds per cubic foot. Determine the force on this part of the concrete form.