Definition: A function **F** is an antiderivative of f on an interval **I** if $F'(x) = f(x)$ for all x in **I**.

Example:

$G(x) = F(x) + C$ represents all antiderivatives of **f**.

- **C** is called the constant of integration.
- The family of functions **G** are called the general antiderivatives of f.
- $G(x) = F(x) + C$ is the general solution of the differential equation $G'(x) = f(x)$.

Definition: A differential equation in x and y is an equation that involves x, y and the derivatives of y.

examples of differential equations:

When solving $\frac{dy}{dx} = f(x)$, we can also write $dy = f(x)dx$.

The process of finding antiderivatives is called either antidifferentiation or indefinite integration.

Notation:

Basic Rules:

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2. Verify $\int \left(4x^3 - \frac{1}{x^2}\right)dx = x^4 + \frac{1}{x} + C$ by showing that the derivative of the right side is equal to the integrand of the left side.

12. $\int x\left(x^2 + 3\right)dx$

14. $\int \frac{1}{(3x)^2}dx$

24. $\int \left(\sqrt[4]{x^3} + 1\right)dx$

28. $\int \frac{x^2 + 2x - 3}{x^4}dx$

34. $\int 3dt$

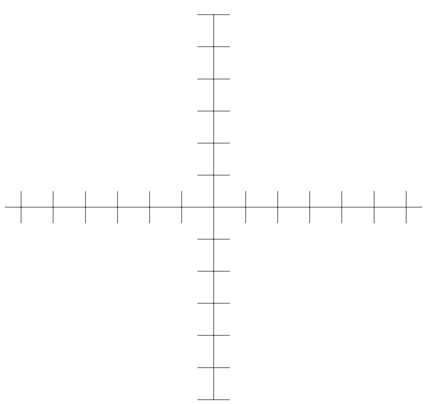
36. $\int \left(t^2 - \sin t\right)dt$

40. $\int \sec y(\tan y - \sec y)dy$

42. $\int \frac{\cos x}{1-\cos^2 x}dx$

48. Find the equation for y, given the derivative and the indicated point on the curve.

$$\frac{dy}{dx} = 2(x-1)$$



60. Solve the differential equation.

$$f''(x) = x^2, \quad f'(0) = 6, \quad f(0) = 3$$

62. Solve the differential equation.

$$f''(x) = \sin x, \quad f'(0) = 1, \quad f(0) = 6$$

68. Show that the height above ground of an object thrown upward from a point s_0 feet above the ground with an initial velocity of v_0 feet per second is given by the function $f(t) = -16t^2 + v_0 t + s_0$.

72. The Grand Canyon is 1800 meters deep at its deepest point. A rock is dropped from the rim above this point. Express the height of the rock as a function of the time *t* in seconds. How long will it take the rock to hit the canyon floor? (Use $a(t) = -9.8$ meters per second per second as the acceleration due to gravity. Ignore air resistance.)

74. With what initial velocity must an object thrown upward (from a height of 2 meters) to reach a maximum height of 200 meters? (Use $a(t) = -9.8$ meters per second per second as the acceleration due to gravity. Ignore air resistance.)

80. A particle, initially at rest, moves along the x-axis such that its acceleration at time **t>0** is given by $a(t) = \cos t$. At the time t=0, its position is x=3.

a. Find the velocity and position function for the particle.

b. Find the values of *t* for which the particle is at rest.

82. A car traveling at 45 miles per hour is brought to a stop, at constant deceleration, 132 feet from where the brakes are applied.

a. How far has the car moved when its speed has been reduced to 30 miles per hour?

b. How far has the car moved when its speed has been reduced to 15 miles per hour?

c. Draw the real number line from 0 to 132, and plot the points found in parts a and b. What can you conclude?

Section 4.2 AREA UNDER A CURVE & SIGMA NOTATION

Recall from previous classes the following formulas for arithmetic and geometric series and sequences.

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}\left(a_1 + a_n\right) = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$a_n = a_1 r^{n-1}$$

$$S_n = \frac{a_1 - a_1 r^n}{1-r} = \frac{a_1 - a_n r}{1-r}$$

Ex. Find the sum: $\sum_{k=2}^{5}(k+1)(k-3)$

8. Express $\frac{5}{1+1} + \frac{5}{1+2} + \frac{5}{1+3} + \ldots + \frac{5}{1+15}$ using sigma notation:

16. Evaluate: $\sum_{i=1}^{15}(2i-3)$

Here are some basic properties of sigma notation.

1. $\sum_{i=1}^{n} k a_i = k \sum_{i=1}^{n} a_i$
2. $\sum_{i=1}^{n}\left(a_i \pm b_i\right) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

These are some formulas that we will need for our calculations.

1. $\sum_{i=1}^{n} c = cn$
2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

28. Use upper and lower sums to approximate the area of the region using the indicated number of subintervals of equal length.

$y = \sqrt{x} + 2$

<u>Notation</u>:

For inscribed rectangles, the **lower sum** is given by $s(n) = \sum_{i=1}^{n} f(m_i)\Delta x$, where $f(m_i)$ is the minimum value of $f(x)$ in the interval.

For circumscribed rectangles, the **upper sum** is given by $S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$, where $f(M_i)$ is the maximum value of $f(x)$ in the interval.

$\Delta x = \frac{b-a}{n}$, where the interval $[a,b]$ is broken up into n subintervals.

<u>Some limits</u>:

$$\lim_{n\to\infty} s(n) = \lim_{n\to\infty} \Sigma f(m_i)\Delta x = \lim_{n\to\infty} \Sigma f(M_i)\Delta x = \lim_{n\to\infty} S(n)$$

As $n \to \infty$, the limits of both the upper and lower sums are equal.

Ex. Find the limit of $s(n)$ as $n \to \infty$.

$$s(n) = \left(\frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}\right)$$

42. Find a formula for the sum of n terms. Use the formula to find the limit as $n \to \infty$.

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^2 \left(\frac{2}{n}\right)$$

Definition of the Area of a region in the plane:
Let f be continuous and nonnegative on $[a, b]$.
The area of the region bounded by f, the x-axis, and the vertical lines x=a and x=b is given by:

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x, \quad x_{i-1} \le c_i \le x_i$$

It follows from the squeeze theorem that any value c_i in the interval does not affect the limit.

48. Use the limit process to find the area of the region bounded between the graph of $y = 3x - 4$ and the x-axis over the interval $[2,5]$. Sketch the region.

Section 4.3 DEFINITE INTEGRALS AND RIEMANN SUMS

Theorem: If a function is continuous on $[a,b]$, then it is integrable on $[a,b]$.

Theorem: If f is continuous and nonnegative on $[a,b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines **x=a** and **x=b** is given by $A=\int_a^b f(x)dx$

Other Properties:

1. $\int_a^a f(x)dx = 0$
2. $\int_b^a f(x)dx = -\int_a^b f(x)dx$
3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, $a < c < b$
4. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$
5. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

Theorem: If f is integrable and nonnegative on $[a,b]$, then $0 \le \int_a^b f(x)dx$.

Theorem: If f and g are integrable on $[a,b]$ and $f(x) \le g(x)$ for all **x** in $[a,b]$, then $\int_a^b f(x)dx \le \int_a^b g(x)dx$.

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In exercises 13-22, set up a definite integral that yields the area of the given region. (Do not evaluate the integral.

16. $f(x) = x^2$

22. $f(y) = (y-2)^2$

In exercises 23-32, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral (a>0 and r>0).

24. $\int_{-a}^{a} 4dx$

32. $\int_{-r}^{r} \sqrt{r^2 - x^2}\,dx$

42. Given $\int_0^3 f(x)dx = 4$ and $\int_3^6 f(x)dx = -1$, find

a. $\int_0^6 f(x)dx$

b. $\int_6^3 f(x)dx$

c. $\int_3^3 f(x)dx$

d. $\int_3^6 -5f(x)dx$